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Mark Scheme (Results)
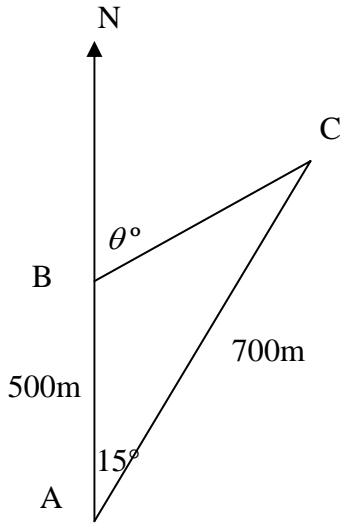# January 2008

GCE

GCE Mathematics (6664/01)

A PEARSON COMPANY

January 2008
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1. a)i) ii)	$f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8 \quad ; \quad = 5$ $f(-2) = (-8 - 8 + 8 + 8) = 0 \qquad$ (B1 on Epen, but A1 in fact) M1 is for attempt at either f(3) or f(–3) in (i) or f(–2) or f(2) in (ii).	M1; A1 A1　(3)
(b)	$[(x + 2)](x^2 - 4x + 4) \qquad$ (= 0 not required) [must be seen or used in (b)] $(x + 2)(x - 2)^2 \qquad$ (= 0) 　(can imply previous 2 marks) Solutions: $x = 2$ or -2 (both) or (–2, 2, 2)　　A1 (4)	M1 A1 M1 **[7]**
Notes: (a)	No working seen: Both answers correct scores full marks 　　　　　　　　One correct ;M1 then A1B0 or A0B1, whichever appropriate. <u>Alternative (Long division)</u> **Divide by** (x – 3) OR $(x + 2)$ to get $x^2 + ax + b,$ 　*a* may be zero 　　[M1] $x^2 + x - 1$ and $+5$ seen 　　i.s.w. (or "remainder = 5")　　[A1] $x^2 - 4x + 4$ and 0 seen 　　　　(or "no remainder")　　[B1]	
(b)	First M1 requires division by a found factor ; e.g $(x + 2),$ $(x - 2)$ or what candidate thinks is a factor to get $(x^2 + ax + b),$ 　*a* may be zero. First A1 for 　$[(x + 2)] (x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$ Second M1:attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x + c)(x + d),$ where $|cd| = |b|.$] **N.B**. Second A1 is for solutions, not factors <u>*Alternative (first two marks)*</u> $(x + 2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare 　　　　　　with 　$x^3 - 2x^2 - 4x + 8 = 0$ to find *b* and *c*.　[M1] 　　　　　　　　$b = -4,\ c = 4$ 　　　　　　[A1] <u>*Method of grouping*</u> $x^3 - 2x^2 - 4x + 8 = x^2(x - 2),4(x \pm 2)$ M1; $= x^2(x - 2) - 4(x - 2)$ A1 　　　　　　　　　　　　　$[= (x^2 - 4)(x - 2)] = (x + 2)(x - 2)^2$ M1 　　Solutions: 　$x = 2,\ x = -2$ both 　A1	
2. (a)	Complete method, using terms of form ar^k, to find r [e.g. **Dividing** $ar^6 = 80$ by $ar^3 = 10$ to find r ; $r^6 - r^3 = 8$ is M0] 　$r = 2$	M1 A1　(2)
(b)	Complete method for finding a [e.g. Substituting value for *r* into equation of form $ar^k = 10$ or 80 and finding a value for *a.*]	M1

		$(8a = 10)$ $\qquad a = \dfrac{5}{4} = 1\dfrac{1}{4}$ (equivalent single fraction or 1.25)	A1 (2)

(c)

Substituting their values of *a* and *r* into **correct** formula for sum.   M1

$$S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1) \quad (= 1310718.75) \qquad 1\,310\,719 \text{ (only this)} \qquad \text{A1 (2) } \mathbf{[6]}$$

Notes:

(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$,
    A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these)
    (M mark can be implied from numerical work, if used correctly)

(b) M1: Allow for numerical approach: e.g. $\quad \dfrac{10}{r_c^{\,3}} \quad \leftarrow \quad \dfrac{10}{r_c^{\,2}} \quad \leftarrow \quad \dfrac{10}{r_c} \leftarrow 10$

In (a) and (b) correct answer, with no working, allow both marks.
(c) Attempt 20 terms of series and add is M1 (correct last term 655360)
    If formula **not** quoted, errors in applying their *a* and/or *r* is M0
    Allow full marks for correct answer with no working seen.

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3. (a)

$$\left(1 + \frac{1}{2}x\right)^{10} = 1 + \binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3 \qquad \text{M1 A1}$$

$$= 1 + 5x \,; + \frac{45}{4}\text{ (or 11.25)}x^2 + 15x^3 \text{ (coeffs need to be these, i.e, simplified) A1; A1 (4)}$$

[Allow A1A0, if totally correct with unsimplified, single fraction coefficients)

(b)

$$(1 + \tfrac{1}{2}\times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4}\,or\,11.25)(0.01)^2 + 15(0.01)^3 \qquad \text{M1 A1}\surd$$

$$= 1 + 0.05 + 0.001125 + 0.000015$$
$$= 1.05114 \qquad \text{cao} \qquad \text{A1 (3) } \mathbf{[7]}$$

Notes:

(a) For M1 first A1: Consider underlined expression only.
    M1 Requires correct structure for at least two of the three terms:
    (i) Must be attempt at binomial coefficients.
    (ii) Must have increasing powers of x,
    (iii) May be listed, need not be added; *this applies for all marks.*

First A1: Requires all three correct terms but need not be simplified, allow
1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of ½ x
Second A1: Consider as B1 for $1 + 5x$

(b) For M1: Substituting their (0.01) into their (a) result
    First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a)
    Answer with no working scores no marks (calculator gives this answer)

4.	(a)	$3 \sin^2 \theta - 2 \cos^2 \theta = 1$		
		$3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (M1: Use of $\sin^2 \theta + \cos^2 \theta = 1$)	M1	
		$3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$		
		$5 \sin^2 \theta = 3$ cso **AG**	A1	(2)
	(b)	$\sin^2 \theta = \dfrac{3}{5}$, so $\sin \theta = (\pm)\sqrt{0.6}$	M1	
		Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -$ (may be implied by later work) M1		
		$\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first M1 only)	A1	
		$\theta \ (= 180° - 50.7685_c °)$; = 129.23…° awrt 129.2°	M1; A1 √	
		[f.t. dependent on first M and 3rd M]		
		$\sin \theta = - \sqrt{0.6}$		
		$\theta = 230.785°$ and $309.23152°$ awrt 230.8°, 309.2° (both)	M1A1	(7)
				[9]
Notes:		(a) N.B: **AG**; need to see at least one line of working after substituting $\cos^2 \theta$.		
		(b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ		
		Second M1: Considering the – value for $\sin \theta$. (usually later)		
		First A1: Given for awrt 50.8°. Not dependent on second M.		
		Third M1: For $(180 - 50.8_c)°$, need not see written down		
		Final M1: **Dependent** on second M (but may be implied by answers)		
		For $(180 + \text{candidate's } 50.8)°$ or $(360 - 50.8_c)°$ **or equiv.**		
		Final A1: Requires both values. (no follow through)		
		[Finds $\cos^2 \theta = k$ (k = 2/5) and so $\cos \theta = (\pm)$...M1, then mark equivalently]		

5.	Method 1 (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of 3b for a (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on correctly derived $\log_3$ p= q e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{\frac{1}{2}}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	Method 2 (Working with two equations in $\log_3$a and $\log_3$b)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1\frac{1}{2}$, $\log_3 b = \frac{1}{2}$]	M1
	Using base correctly to find a or b	M1
	Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1**[6]**
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on Method 1, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of $a = 3b$, (iii) Third M1: requires using base correctly on correctly derived $\log_3$ p= q	

6.

N

C

$\theta°$

B

700m

500m

15°

A

$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ M1 A1
   (= 63851.92…)
$BC = 253$ awrt   A1   (3)

(a)

$\dfrac{\sin B}{700} = \dfrac{\sin 15}{\text{candidate's } BC}$ M1

$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°)   dep   M1

(b)

$\theta = 180°$ − candidate's angle B (Dep. on first M only, B can be acute)   M1
$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9)   A1 (4) **[7]**
    [46 needs to be from correct working]

Notes:

(a) If use cos 15° = ….., then A1 not scored until written as BC² = … correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC
Finding value for *BX* and *CX* and using Pythagoras   M1
$BC^2 = (500 \sin 15°)^2 + (700 - 500 \cos 15°)^2$ A1
$BC = 253$ awrt   A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))

(i) $\cos B = \dfrac{500^2 + \text{candidate's} BC^2 - 700^2}{2 \text{x} 500 \text{x candidate's} BC}$ or $700^2 = 500^2 + BC_c^{\ 2} - 2x500xBC_c$ M1
    Finding angle B M1,   then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)
      $\tan CBX = \dfrac{700 - valueforAX}{valueforBX}$ M1
      Finding value for $\angle CBX$ ($\approx 59°$)   M1
      $\theta = [180° - (75° + candidate's \angle CBX)]$ M1
(iii) Using sine rule (or cos rule) to find *C* first:
     Correct use of sine or cos rule for C M1,   Finding value for C M1
Either $B = 180° - (15° + $ candidate's $C)$ or $\theta = (15° + $ candidate's $C)$ M1
(iv) $700 \cos 15° = 500 + BC \cos \theta$ M2 {first two Ms earned in this case}
     Solving for θ; $\theta = 45.8$ (allow 46 or5.7, 45.8, 45.9 M1;A1

7	(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$)	B1	(1)
		or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]		
	(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = ..	M1	
		$x = 4$ (and $x = 0$)	A1	
		Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	A1	(3)
	(c)	(Area =) $\int_{(0)}^{(4)}(6x - x^2)\,dx$ Limits not required	M1	
		Correct integration $3x^2 - \dfrac{x^3}{3}$ $(+ c)$	A1	
		Correct use of correct limits on their result above (see notes on limits)	M1	
		$[\text{"}\ 3x^2 - \dfrac{x^3}{3}\text{"}]^4 - [\text{"}\ 3x^2 - \dfrac{x^3}{3}\text{"}]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}\ ]$		
		Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1)	A1	
		Shaded area = $\pm$ (area under curve – area of triangle) applied correctly	M1	
		$(= 26\frac{2}{3} - 16)$ $= 10\frac{2}{3}$ (awrt 10.7)	A1 (6)	**[10]**

Notes	(b) In scheme first A1: need only give $x = 4$	
	If *verifying approach* used:	
	Verifying (4,8) satisfies both the line and the curve M1(attempt at both),	
	Both shown successfully A1	
	For final A1, (0,0) needs to be mentioned ; accept " clear from diagram"	
	(c) **Alternative** Using Area $= \pm \int_{(0)}^{(4)}\{(6x - x^2);\ -2x\}\,dx$ approach	
	(i) If candidate integrates separately can be marked as main scheme	
	If combine to work with $= \pm \int_{(0)}^{(4)}(4x - x^2)\,dx$, **first** M mark and **third** M mark	
	$= (\pm)\,[\,2x^2 - \dfrac{x^3}{3}\ (+ c)\,]$ A1,	
	Correct use of correct limits on their result **second** M1,	
	Totally correct, unsimplified $\pm$ expression (may be implied by correct ans.) A1	
	$10\frac{2}{3}$ A1 [Allow this if, having given - $10\frac{2}{3}$, they correct it]	
	M1 *for correct use of correct limits*: Must substitute correct limits for their	
	strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$	
	If a long method is used, e,g, finding three areas, this mark only gained for	
	correct strategy and all limits need to be correct for this strategy.	
	Use of trapezium rule: M0A0MA0,possibleA1for triangle	
	M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

8	(a)	$(x - 6)^2 + (y - 4)^2 = $; 3^2	B1; B1 (2)
	(b)	Complete method for MP: $ = \sqrt{(12-6)^2 + (6-4)^2}$	M1
		$= \sqrt{40}$ $(= 6.325)$	A1
		[These first two marks can be scored if seen as part of solution for (c)]	
		Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
		e.g. $\cos\theta = \dfrac{MT}{MP} = \dfrac{3}{candidate's\sqrt{40}}$ $(= 0.4743)$ $(\theta = 61.6835^\circ)$ [If TP = 6 is used, then M0]	
		$\theta = 1.0766$ rad **AG**	A1 (4)
	(c)	Complete method for area TMP; e.g. $= \dfrac{1}{2} \times 3 \times \sqrt{40}\sin\theta$	M1
		$= \dfrac{3}{2}\sqrt{31}$ $(= 8.3516..)$ allow awrt 8.35	A1
		Area (sector)MTQ $= 0.5 \times 3^2 \times 1.0766$ $(= 4.8446…)$	M1
		Area TPQ = candidate' s $(8.3516.. - 4.8446..)$	M1
		$= 3.507$ awrt [Note: 3.51 is A0]	A1 (5) **[11]**
Notes		(a) Allow 9 for 3^2.	
		(b) First M1 can be implied by √ 40	
		For second M1: May find TP = $\sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either	
		$\sin\theta = \dfrac{TP}{MP} = \dfrac{\sqrt{31}}{\sqrt{40}}$ $(= 0.8803...)$ or $\tan\theta = \dfrac{\sqrt{31}}{3}$ $(1.8859..)$ or cos rule	
		NB. Answer is given, but allow final A1 if all previous work is correct.	
		(c) First M1: (alternative) $\dfrac{1}{2} \times 3 \times \sqrt{40-9}$	

9	(a)	(Total area) $= 3xy + 2x^2$	B1
		(Vol:) $\qquad x^2 y = 100 \qquad\qquad (y = \dfrac{100}{x^2}, xy = \dfrac{100}{x})$	B1
		Deriving expression for area in terms of x only	M1
		(Substitution, or clear use of, y or xy into expression for area)	
		(Area =) $\quad \dfrac{300}{x} + 2x^2 \qquad\qquad$ **AG**	A1 cso (4)
	(b)	$\dfrac{dA}{dx} = -\dfrac{300}{x^2} + 4x$	M1A1
		Setting $\dfrac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1	
		[$x^3 = 75$]	
		$\qquad\qquad\qquad x = 4.2172 \quad$ awrt 4.22 $\quad$ (allow exact $\sqrt[3]{75}$)	A1 (4)
	(c)	$\dfrac{d^2A}{dx^2} = \dfrac{600}{x^3} + 4 = $ positive $\qquad\qquad$ therefore minimum	M1A1 (2)
	(d)	Substituting found value of x into (a)	M1
		(Or finding y for found x and substituting both in $3xy + 2x^2$)	
		$[y = \dfrac{100}{4.2172^2} = 5.6228]$	
		Area = 106.707 $\qquad$ awrt 107	A1 (2)
			[12]
Notes		(a) First B1: Earned for correct unsimplified expression, isw.	
		(c) For M1: Find $\dfrac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive"	
		$\qquad$ A1: Candidate's $\dfrac{d^2A}{dx^2}$ must be correct for their $\dfrac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found)	
		<u>Alternative:</u> M1: Find value of $\dfrac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign	
		$\qquad$ A1: Indicate sign change of negative to positive for $\dfrac{dA}{dx}$, and conclude minimum.	
		OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with "107" A1: Both values greater than "$x = 107$" and conclude minimum.	
		Allow marks for (c) and (d) where seen; even if part labelling confused.	